

The impact of initial density profile on protoplanetary disc evolution simulation *

G.G.Lazareva, A.V.Snytnikov, V.A.Vshivkov

Abstract

In the simulation of protoplanetary disc with a power law density profile a disc instability is detected. The instability arises only with a power law profile and is affected by power index. Thus the impact of initial density profile is large within the employed numerical model.

1 Introduction

Density profile is the dependence of disc surface density on radius. As it is pointed in [1], at present the true density profiles in protoplanetary discs are unknown. Nevertheless in most works density profile is thought to be a power law with different index. The index should be -1.0 to correspond the observation data [2]. In simulation various indexes in the range from -0.5 to -2.5 are being used [1], [3], [5].

MMSN model (Minimal Mass Solar Nebula) was proposed in [4]. The density of solid particles in this model was obtained by imagining grinding up the planets, distributing their mass smoothly with radius and adding up enough gas to make Solar composition. The resulting density profile is the following:

$$\sigma(r) = \sigma_1 \left(\frac{r}{1 \text{ a.u.}} \right)^{-1.5}$$

Here $\sigma_1 = 1700 \text{ g/cm}^2$, $1 \text{ a.u.} = 1.5 \times 10^{11} \text{ m}$. The ratio of gas and solid particles mass in MMSN model is the same as in the Solar System (100:1). Unfortunately, this model is scarcely applicable to extrasolar planetary systems as it follows from observation of T Tauri stars [2]. Therefore the simulation of protoplanetary discs with different initial density profiles is conducted [1], [3].

The aim of the present work is to simulate the evolution of the protoplanetary disc with different initial density profiles and to compare the results with the known simulations of planet system formation.

*The present work was supported by RAS Presidium Program 18.1 "Biosphere genesis and evolution", RFBR grant 05-01-00665, Dutch-Russian NWO-GRID project, contract NWO-RFBS 047.016.007 and Dutch-Russian NWO-Plasma project, contract NWO-RFBS 047.016.018 and also by the Grant of Rosobrazovanie, contracts PHIL.2.2.1.1.3653 and PHIL.2.2.1.1.1969.

2 Simulation

Two density profiles were taken as initial profiles for computational experiments: solid body profile (σ_S) and power law profile (σ_P):

$$\sigma_S = \sigma_1 \sqrt{1 - \left(\frac{r}{R_D}\right)^2}, \quad r < R_D,$$

$$\sigma_P = \sigma_1 r^\alpha, \quad \alpha = -0.5, \dots, -1.5.$$

Here R_D is the disc radius and the value of σ_1 is set for the disc mass to be equal to the given value.

The computational experiments were conducted in size-less variables in order to decrease round-off errors. The following quantities were chosen as basic characteristic parameters for transition to size-less variables:

- distance from the Sun to the Earth $R_0 = 1.5 \cdot 10^{11}$ m;
- mass of the Sun $M_\odot = 2 \cdot 10^{30}$ kg;
- gravitational constant $G = 6.672 \cdot 10^{-11} \cdot \text{m}^2/\text{kg}^2$.

Corresponding characteristic values of the particle velocity (V_0), time (t_0), potential (Φ_0) and surface density (σ_0) are written as

$$V_0 = \sqrt{\frac{GM_\odot}{R_0}} = 30 \text{ km/s},$$

$$t_0 = \frac{R_0}{V_0} = 5 \cdot 10^6 \text{ s} = 1/6 \text{ year},$$

$$\Phi_0 = V_0^2 = \frac{GM_\odot}{R_0},$$

$$\sigma_0 = \frac{M_\odot}{R_0^2}.$$

In the following text all the parameters are given in size-less units.

The ratio of central body mass, gas mass and dustsolid particles mass was set according to the MMSN model: central body mass $M_\odot = 10.0$, gas mass was $M_G = 1.0$ and solid particles mass $M_P = 0.01$. Both solid particles and gas were given the Keplerian velocity v_K (in any point of the disc the centrifugal force is equal to gravitational one):

$$\frac{\sigma v_K^2}{r} = -\frac{\partial \Phi}{\partial r}$$

Other parameters: initial disc radius $R_D = 2.0$, radius of computational domain $R_M = 9.0$.

The dynamics of the solid particle component of protoplanetary disc is described by the Vlasov-Liouville kinetic equation. In the following text dustsolid particles will be called simply particles. To consider motion of the gas component the equations of gas dynamics are employed. The gravitational field is determined by Poisson equation.

If we employ the collisionless approximation of the mean self-consistent field, then Vlasov-Liouville kinetic equation is written in the following form

$$\frac{\partial f}{\partial t} + \vec{v} \nabla f + \vec{a} \frac{\partial f}{\partial \vec{v}} = 0,$$

where $f(t, \vec{r}, \vec{v})$ is the time-dependent one-particle distribution function along coordinates and velocities, $\vec{a} = -\nabla \Phi + \frac{\vec{F}_{fr}}{m}$ is the acceleration of unit mass particle, \vec{F}_{fr} is the friction force between gas and particle components of the medium. Gravitational potential Φ could be divided into two parts:

$$\Phi = \Phi_1 + \Phi_2$$

where Φ_1 presents the potential of protostar. The second part of potential Φ_2 is determined by the additive distribution of the moving particles and gas. Φ_2 satisfies the Poisson equation

$$\Delta \Phi_2 = 4\pi G \Sigma \rho$$

In the case of a flat disc the bulk density of the mobile media $\Sigma \rho = \rho_{part} + \rho_{gas}$ is equal to zero (ρ_{part} is the particle density ρ_{gas} is the gas density). At the disc with the surface density σ there is a shear of the normal derivative of potential. This shear gives a boundary condition for the normal derivative of potential Φ_2 :

$$\frac{\partial \Phi_2}{\partial z} = 2\pi G \sigma$$

The equations of gas dynamics take the following form:

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0$$

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \nabla) \vec{v} \right] = -\nabla p + \vec{F}$$

$$\frac{\partial E}{\partial t} + (\vec{v} \nabla) E = -\nabla (p \vec{v}) + Q + (\vec{F}, \vec{v}) - \nabla W$$

where $E = \left(\varepsilon + \frac{v^2}{2} \right)$ is the density of gas full energy, $\varepsilon = \varepsilon(\rho, T)$ is the internal energy of gas, $p = p(\rho, T)$ is the pressure of gas, $\vec{W} = \nabla T$ is the heat flow, Q is the increase of energy due to chemical reactions and radiation. \vec{F} is the external force which is defined by the following expression

$$\vec{F} = \rho \nabla \Phi - k_{fr} (\vec{u} - \vec{v})$$

here k_{fr} is the coefficient of friction between gas and particle components of the disc, \vec{u} is the particle velocity, \vec{v} is the gas velocity. In the case of the flat disc the form of equation remains the same with the only exclusion: bulk density ρ

is replaced with surface density σ . In this paper we shall consider only the flat disc model.

Vlasov-Liouville equation is solved by Particles-in-Cells method. To solve the equations of gas dynamics Fluids-in-Cells method is employed. Poisson equation is solved by a combination of FFT and SOR methods. A detailed description of the code could be found in [6].

In the computational experiments the cylindrical coordinate system was used, grid size is $N_R \times N_\varphi \times N_Z = 300 \times 256 \times 100$. The experiments were conducted with the MVS-1000M multicomputer of the Siberian Supercomputer Centre (32 Alpha21264 processors, 833 MHz).

3 Results

The most interesting result is that the disc with the massive central body is unstable when the initial density profile satisfies the power law. Instability here is the loss of axial symmetry in the central are of the disc, as it is shown in figure 1: a group of dens gaseous clumps is formed around the central body.

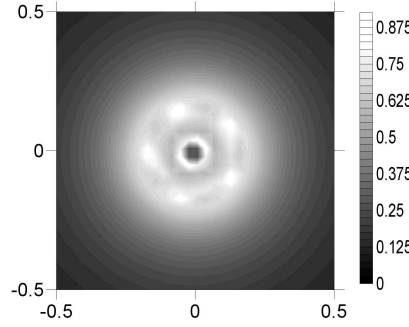


Figure 1: Gas density in the central are of the disc.

It is important because usually the massive central body suppresses all the angular instabilities [7]. Furthermore, this instability arises for the disc with power law profile and does not arise for the disc with solid body profile.

In order to show the development of instability the Fourier analysis of gas density was conducted along angular direction:

$$\sigma(r, \varphi, t) = \sum_{k=1}^{N_\varphi-1} S_k(r, t) \cos\left(\frac{2\pi}{N_\varphi} k \varphi\right)$$

$$S_k(r, t) = \frac{1}{N_\varphi} \sum_{k=1}^{N_\varphi-1} S_k(r, t) \cos\left(\frac{2\pi}{N_\varphi} k \varphi\right)$$

$$S_{MAX}(t) = \max S_k(r, t), \quad 0 < r < R_M, \quad 1 \leq k < N_\varphi$$

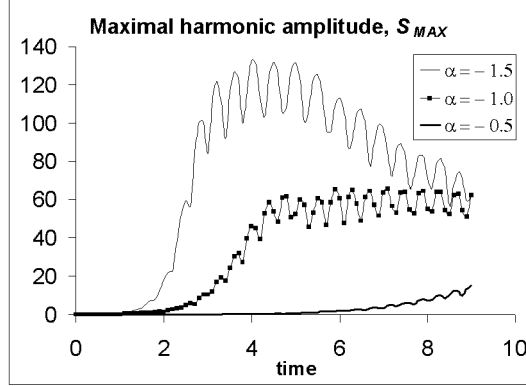


Figure 2: Instability development in the disc with power law profile for various indexes α .

Figure 2 shows maximal harmonic amplitude S_{MAX} depending on time with various power indexes α .

It should be noticed that zero harmonic ($k = 0$) is not displayed in figure 2 because this harmonic shows instabilities that do not break axial symmetry. One can see from figure 2 that the harmonic amplitude increases sufficiently with time.

Now let us consider the behavior of disc with various indexes α . Figure 2 shows that with lesser values of α the instability develops faster and harmonics have greater amplitude. This fact correspond the results of [1]: in their simulations discs with lower α formed planets earlier and the planets had greater mass.

It is also stated in [1] and [3] that for steeper profiles (lower values of α), the terrestrial planets are more massive. This result is supported by the figure 3 that shows average grain particle density depending on radius.

4 Conclusion

It follows from the computational experiment that within the employed model of the protoplanetary disc the impact of the initial density profile is large. The power law profile leads to the development of the angular instability, while the disc with solid body profile remains stable. Moreover, the instability develops faster with lower values of power index. The decrease of the power index also leads to increase of amplitude of unstable harmonics and to higher mass of gas and grain clumps.

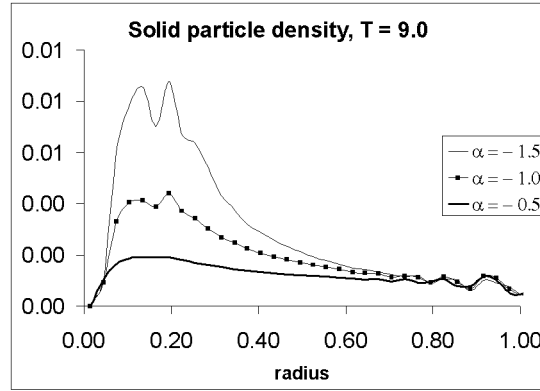


Figure 3: Gas density in the central are of the disc.

References

- [1] S.N.Raymond, T.Quinn, J.I.Lunine. Terrestrial planet formation in disks with varying surface density profiles. //Astrophysical Journal, 632:670-676, 2005.
- [2] P.D'Alessio, N.Calvet, L.Hartmann, S.Lizano, J.Canto. Accretion disks around young objects. II. Tests of well-mixed models with ISM dust. //Astrophysical Journal, 527:893-909, 1999.
- [3] E.Kokubo, Sh.Ida. Formation of protoplanet system and diversity of planetary systems. // Astrophysical Journal, 581:666-680, 2002.
- [4] C.Hayashi. //Prog.Theor.Phys.Suppl., 70, 35, 1981.
- [5] W.K.M.Rice, K.Wood, B.A.Whitney, J.E.Bjorkman. Constraints on a planetary origin for the gap in the protoplanetary disc of GM Aurigae. //Mon.Not.R.Astron.Soc. 342, 79-85, 2003.
- [6] E.A.Kuksheva, V.E.Malyshkin, S.A.Nikitin, A.V.Snytnikov, V.N.Snytnikov, V.A.Vshivkov. Numerical Simulation of Self-Organisation in Gravitationally Unstable Media on Supercomputers. // PaCT-2003 proceedings , LNCS 2763, pp.354-368, 2003.
- [7] BERTIN G. // Dynamics of Galaxies. Cambridge University Press, 2000.